5.4 Slope as a Rate of Change

Asafa Powell of Jamaica set the men's 100-m world record in Athens, Greece, on June 14, 2005. He ran 100 m in 9.77 s. His average speed can be found by dividing the distance by his time.

$$\text{Average speed} = \frac{100}{9.77}$$
$$\doteq 10.2$$

Asafa's average speed was about 10.2 m/s. This means that, on average, he covered a distance of 10.2 m per second of the race. Speed is an example of a **rate of change**, because it is a rate that refers to the change in distance relative to the change in time.

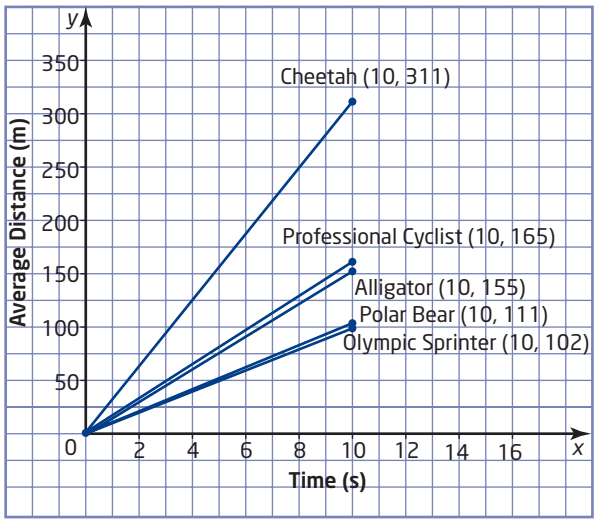
rate of change

- a change in one quantity relative to the change in another quantity

Investigate

How can you find a rate of change from a graph?

The graph shows the average distance, in metres, that each animal or person can run in 10 s.



1. What do you think is meant by average distance?

2. Visually compare the steepness of each graph. Determine the slope of each graph. Rank the slopes from least to greatest.

3. Calculate the speed of each animal or person as $\frac{\text{distance}}{\text{time}}$. Rank the speeds from least to greatest.

4. **Reflect** Describe how the rate of change relates to the graph of a relation.

Example 1 Speed

Sarah is on the soccer team and runs every morning before school. One day, she ran 5 km in 20 min.

a) Calculate the rate of change of Sarah's distance from her starting point.

b) Graph Sarah's distance as it relates to time.

c) Explain the meaning of the rate of change and how it relates to the graph.

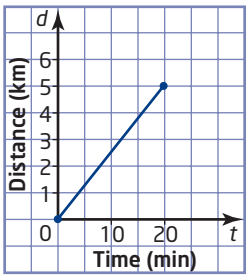
Solution

a) The rate of change is the distance travelled over the elapsed time.

$$\text{rate of change} = \frac{\text{change in distance}}{\text{change in time}}$$
$$= \frac{5}{20}$$
$$= 0.25$$

The rate of change is 0.25 km/min.

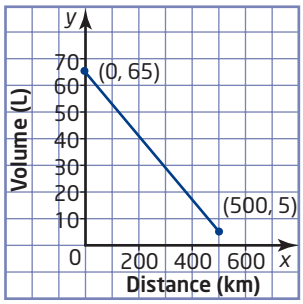
b)



c) The rate of change is Sarah's average running speed. It is also the slope of the graph.

Example 2 Fuel Consumption

The graph shows the volume of gasoline remaining in a car's tank.

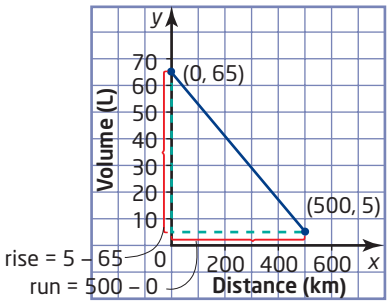


a) Calculate the slope of the graph.

b) Interpret the slope as a rate of change.

Solution

a) Finding the rise and run by counting grid squares may not always be practical. You can also find the rise and the run from the coordinates of two points on the graph.



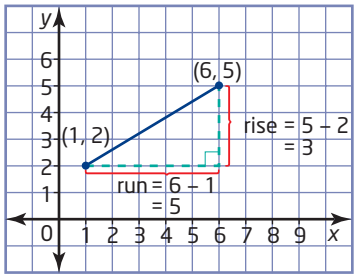
$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{5 - 65}{500 - 0}$$
$$= \frac{-60}{500}$$
$$= -0.12$$

I can subtract the *y*-values to get the rise. I can subtract the *x*-values in the same order to get the run. I'll subtract the coordinates of the left point from the coordinates of the right point.

b) The rate of change of the volume of gasoline is -0.12 L/km. The car uses an average of 0.12 L of gasoline per kilometre driven. The rate of change is negative because the volume of gasoline in the tank is decreasing.

Key Concepts

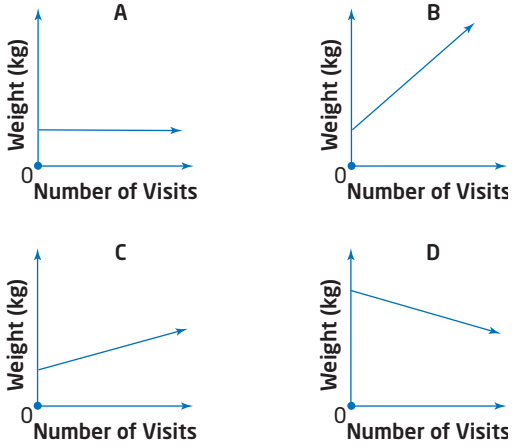
- Rate of change is the change in one quantity relative to the change in another.
- A rate of change requires units, such as kilometres per hour.
- When a relation is graphed, the slope describes the rate of change.
- To find the slope of a line segment joining two points, subtract the y-values to get the rise and subtract the x-values in the same order to get the run.

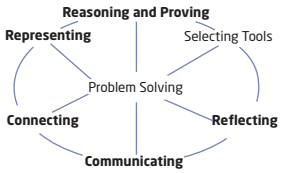


Communicate Your Understanding

C1 A car travelled 400 km in 5 h. Ahmed calculated the speed as 80 km/h but, when he graphed the relation, he calculated a slope of 0.0125. What do you think Ahmed did incorrectly?

C2 When Carlos goes to the gym, he likes to lift weights. The gym has produced graphs illustrating the weight a person should lift over a number of visits. Match each graph with the appropriate situation.





a) Begin with a small weight and slowly increase the weight at a constant rate.

b) Begin with a small weight and lift the same weight each visit.

c) Start with a large weight and slowly decrease the weight lifted each visit.

d) Start with a small weight and rapidly increase the weight at a constant rate each visit.

Practise

For help with questions 1 to 3, see Example 1.

1. The average adult breathes in about 37 L of air every 5 min. What is the rate of change of volume of air?

2. A teenager's heart pumps an average of 7200 L of blood every 24 h. What is the rate of change of volume of blood?

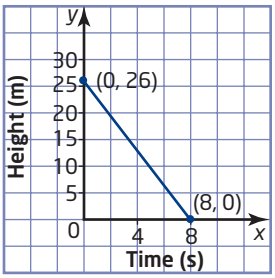
3. A hummingbird can flap its wings an average of 1800 times every 30 s. What is the rate of change of wing flaps?

For help with questions 4 and 5, see Example 2.

4. The graph shows the height above the ground of a rock climber over time.

a) Calculate the slope of the graph.

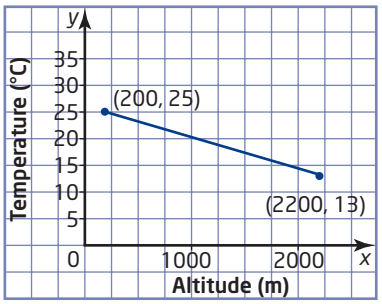
b) Interpret the slope as a rate of change.



5. The graph shows the relationship between temperature and altitude.

a) Calculate the slope of the graph.

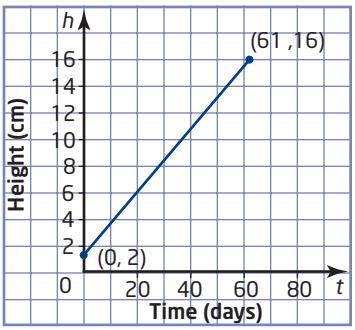
b) Interpret the slope as a rate of change.



Connect and Apply

6. The price of a loaf of bread increased from $1.45 in 2003 to $1.78 in 2006. What is the average price increase per year?

7. The graph shows the height of a plant over a 2-month growth period. Calculate the rate of change per day.



8. The table shows the approximate number of downloads of a freeware program on the Internet over a 2-month period.

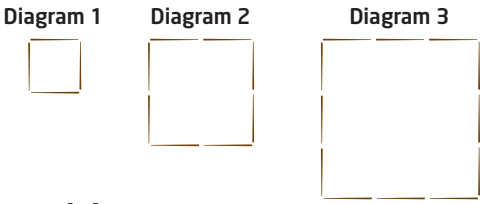
Date	Downloads
Sept 3	52 000
Sept 10	70 000
Sept 17	88 000
Sept 24	106 000
Oct 1	124 000
Oct 8	142 000
Oct 15	160 000
Oct 22	178 000
Oct 29	196 000
Nov 5	214 000

a) Graph the data.

b) Calculate the slope and describe it as a rate of change.

c) Do you think this is a popular piece of software? Why or why not?

9. Chapter Problem



a) Plot a graph relating the number of toothpicks to the diagram number.

b) Calculate the slope of the line through these points.

c) Interpret the slope as a rate of change.

10. From age 12 to 16, girls grow at an average of 8.5 cm/year, while boys grow at an average of 9.5 cm/year. Helen and John are both 12 years old. Helen is 150 cm tall and John is 146 cm tall. Graph their heights on the same grid. When can they expect their heights to be the same?

11. A fire hose can deliver water at a maximum rate of 500 L/min.

a) Plot a graph showing the maximum volume of water that a fire hose can pour onto a fire in time spans of up to 30 min.

b) Suppose two fire hoses are used. How will this affect the slope of the graph?

12. The table shows the minimum volume of water needed to fight a typical fire in rooms of various sizes.

Floor Area (m^2)	Minimum Volume of Water (L)
25	39
50	78
75	117

a) Graph the data in the table.

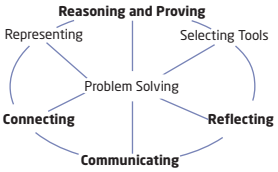
b) Calculate the rate of change.

c) If a fire truck is pumping water at a rate of 200 L/min, how long will it take to put out a fire in a room with a floor area of 140 m^2?

13. A large party balloon is being filled with helium at a constant rate. After 8 s, there is 2.5 L of helium in the balloon.

a) Graph this relation.

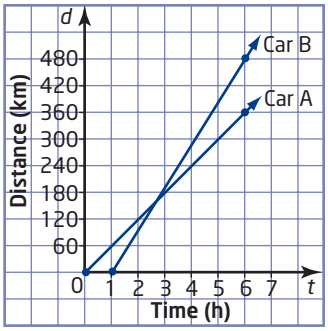
b) The balloon will burst if there is more than 10 L of helium in it. How long will it take to fill the balloon with that much helium? Mark this point on your graph.



14. The distance-time graph shows two cars that are travelling at the same time.

a) Which car has the greater speed, and by how much?

b) What does the point of intersection of the two lines represent?



15. The table shows the number of people who have a university degree in Canada.

Year	Number of People With Degrees (millions)
1990	2.3
1995	3.0
2000	3.7
2005	4.7

a) Graph the data with a broken-line graph.

b) When was the rate of change relatively constant?

c) When was the rate of change different? How was it different?

16. A scuba tank holds 2.6 m^3 of compressed air. A diver at a shallow depth uses about 0.002 m^3 per breath and takes about 15 breaths per minute.

a) How much air will the diver use in 1 min?

b) How long will the air in the tank last at this rate?

c) At a depth of 10 m, the diver is breathing compressed air at 0.004 m^3 per breath. How long will the air last at this depth?

d) At the maximum depth recommended for sport diving, a diver is breathing air at a rate of 0.01 m^3 per breath. How long will the air last at this depth?

17. The table shows the number of people in Canada employed in the tourism industry by year.

a) Is the rate of change constant over the 10-year period?

b) Are the rates of change large or small relative to the total number of jobs? Explain.

Year	Jobs (thousands)
1995	38.8
1996	37.4
1997	37.5
1998	37.9
1999	39.9
2000	41.8
2001	41.7
2002	42.0
2003	41.6
2004	43.1

Achievement Check

18. The fuel efficiency of cars is stated in litres per 100 km. Kim's car has a fuel efficiency of 8 L/100 km. The fuel tank on Kim's car holds 32 L.

a) Graph the relationship between the amount of gasoline remaining in the car's tank and the distance Kim drives. Assume that she started with a full tank.

b) Find the slope of the graph. What does this slope represent?

c) Kim's car uses 25% more gas when she drives in the city. Redraw the graph. Find and interpret the slope for driving in the city.

Extend

19. A store is holding a special clearance sale on a $200 coat. Initially, there is a discount of 5%. Every 2 h, an additional 5% is taken off the latest price.

a) Make a table showing the price over the 16 h the sale is in effect.

b) Graph the price over the 16 h of the sale.

c) Explain the shape of the graph.

20. A cell phone company does not have a monthly fee but charges by the minute. The graph shows the monthly cost of phone calls based on the number of minutes talked.

Describe this cell phone company's rate plan.